Paper Reference(s)

6663/01

Edexcel GCE

Core Mathematics C1

Advanced Subsidiary

Monday 14 January 2013 – Morning

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Pink)	Nil

Calculators may NOT be used in this examination.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C1), the paper reference (6663), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 11 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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1. Factorise completely $x - 4x^3$.

(3)

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2. Express $8^{2x + 3}$ in the form 2^y, stating y in terms of x.

(2)

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3. (i) Express

$$(5 - \sqrt{8})(1 + \sqrt{2})$$

in the form $a + b\sqrt{2}$, where a and b are integers.

(3)

(ii) Express

$$\sqrt{80} + \frac{30}{\sqrt{5}}$$

in the form $c\sqrt{5}$, where c is an integer.

(3)

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4. A sequence $u_1,\ u_2,\ u_3,\ \ldots$, satisfies

$$u_{n+1} = 2u_n - 1, \quad n \geq 1.$$

Given that $u_2 = 9$,

(*a*) find the value of u_3 and the value of u_4 ,

(2)

(*b*) evaluate $\sum_{r=1}^{4} u_r$.

(3)

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5. The line l_1 has equation $y = -2x + 3$.

The line l_2 is perpendicular to l_1 and passes through the point (5, 6).

(*a*) Find an equation for l_2 in the form $ax + by + c = 0$, where a, b and c are integers.

(3)

The line l_2 crosses the x-axis at the point A and the y-axis at the point B.

(*b*) Find the x-coordinate of A and the y-coordinate of B.

(2)

Given that O is the origin,

(*c*) find the area of the triangle OAB.

(2)

6.

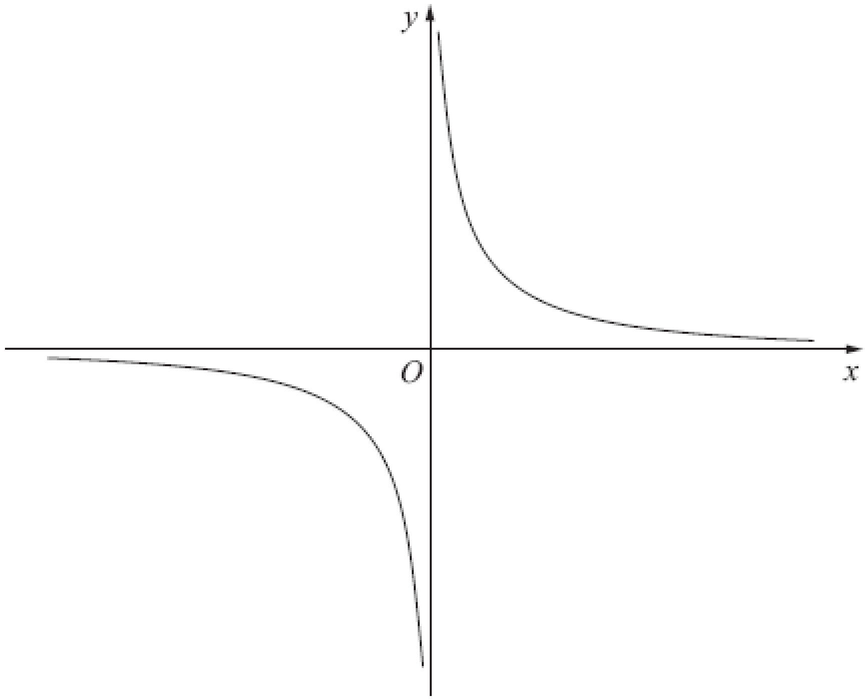

Figure 1

Figure 1 shows a sketch of the curve with equation $y = \frac{2}{x}$, $x \neq 0$.

The curve C has equation $y = \frac{2}{x} - 5$, $x \neq 0$, and the line l has equation $y = 4x + 2$.

(*a*) Sketch and clearly label the graphs of C and l on a single diagram.

On your diagram, show clearly the coordinates of the points where C and l cross the coordinate axes.

(5)

(*b*) Write down the equations of the asymptotes of the curve C.

(2)

(*c*) Find the coordinates of the points of intersection of $y = \frac{2}{x} - 5$ and $y = 4x + 2$.

(5)

7. Lewis played a game of space invaders. He scored points for each spaceship that he captured.

Lewis scored 140 points for capturing his first spaceship.

He scored 160 points for capturing his second spaceship, 180 points for capturing his third spaceship, and so on.

The number of points scored for capturing each successive spaceship formed an arithmetic sequence.

(*a*) Find the number of points that Lewis scored for capturing his 20th spaceship.
(2)

(*b*) Find the total number of points Lewis scored for capturing his first 20 spaceships.
(3)

Sian played an adventure game. She scored points for each dragon that she captured. The number of points that Sian scored for capturing each successive dragon formed an arithmetic sequence.

Sian captured n dragons and the total number of points that she scored for capturing all n dragons was 8500.

Given that Sian scored 300 points for capturing her first dragon and then 700 points for capturing her nth dragon,

(*c*) find the value of n.
(3)

8.
$$\frac{\mathrm{d}y}{\mathrm{d}x} = -x^3 + \frac{4x-5}{2x^3}, \quad x \neq 0.$$

Given that $y = 7$ at $x = 1$, find y in terms of x, giving each term in its simplest form.
(6)

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9. The equation

$$(k+3)x^2 + 6x + k = 5, \text{ where } k \text{ is a constant,}$$

has two distinct real solutions for x.

(*a*) Show that k satisfies

$$k^2 - 2k - 24 < 0.$$
(4)

(*b*) Hence find the set of possible values of k.
(3)

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10. $$4x^2 + 8x + 3 \equiv a(x + b)^2 + c.$$

(*a*) Find the values of the constants a, b and c.

(3)

(*b*) Sketch the curve with equation $y = 4x^2 + 8x + 3$, showing clearly the coordinates of any points where the curve crosses the coordinate axes.

(4)

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11. The curve C has equation

$$y = 2x - 8\sqrt{x} + 5, \quad x \geq 0.$$

(*a*) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$, giving each term in its simplest form.

(3)

The point P on C has x-coordinate equal to $\frac{1}{4}$.

(*b*) Find the equation of the tangent to C at the point P, giving your answer in the form $y = ax + b$, where a and b are constants.

(4)

The tangent to C at the point Q is parallel to the line with equation $2x - 3y + 18 = 0$.

(*c*) Find the coordinates of Q.

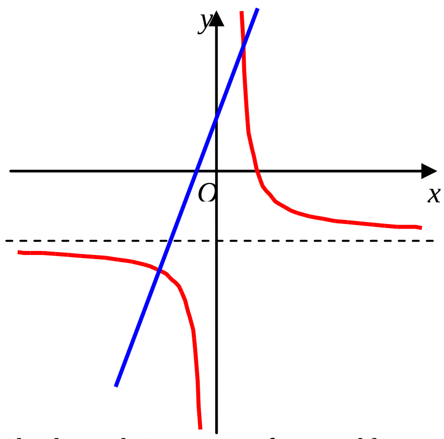
(5)

TOTAL FOR PAPER: 75 MARKS

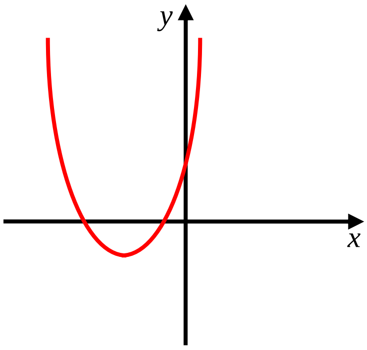
END

Question Number	Scheme	Marks
1.	$x(1-4x^2)$	B1
	Accept $x(-4x^2+1)$ or $-x(4x^2-1)$ or $-x(-1+4x^2)$ or even $4x(\frac{1}{4}-x^2)$ or equivalent quadratic (or initial cubic) into two brackets	M1
	$x(1-2x)(1+2x)$ or $-x(2x-1)(2x+1)$ or $x(2x-1)(-2x-1)$	A1
		[3]
		3 marks
2.	$(8^{2x+3}=\left(2^3\right)^{2x+3})=2^{3(2x+3)}$ or 2^{ax+b} with a = 6 **or** b = 9	M1
	$=2^{6x+9}$ *or* $=2^{3(2x+3)}$ as **final** answer with no errors or $(y=)6x+9$ or $3(2x+3)$	A1
		[2]
		2 marks
3. (i)	$\left(5-\sqrt{8}\right)\left(1+\sqrt{2}\right)$	
	$=5+5\sqrt{2}-\sqrt{8}-4$	M1
	$=5+5\sqrt{2}-2\sqrt{2}-4$ $\sqrt{8}=2\sqrt{2}$, seen or implied at any point.	B1
	$=1+3\sqrt{2}$ $1+3\sqrt{2}$ or $a=1$ and $b=3$.	A1 **[3]**
(ii)	**Method 1:** **Either** $\sqrt{80}+\frac{30}{\sqrt{5}}\left(\frac{\sqrt{5}}{\sqrt{5}}\right)$ **Method 2:** **Or** $\left(\frac{\sqrt{400}+30}{\sqrt{5}}\right)\frac{\sqrt{5}}{\sqrt{5}}$ **Method 3:** $\sqrt{80}+\frac{\sqrt{900}}{\sqrt{5}}=\sqrt{80}+\sqrt{180}$	M1
	Method 1: $=4\sqrt{5}+\ldots$ Method 2: $=\left(\frac{20+..}{..}\right)\frac{..}{..}$ Method 3: $=4\sqrt{5}+\ldots$	B1
	Method 1: $=4\sqrt{5}+6\sqrt{5}$ Method 2: $=\left(\frac{50\sqrt{5}}{5}\right)$ $=10\sqrt{5}$ Method 3: $=4\sqrt{5}+6\sqrt{5}$	A1
		[3]
		6 marks

Question Number	Scheme		Marks
4. (a)	$u_2 = 9,\ u_{n+1} = 2u_n - 1,\ \ n \ldots 1$		
	$u_3 = 2u_2 - 1 = 2(9) - 1 \quad (=17)$	$u_3 = 2(9) - 1.$ Can be implied by $u_3 = 17$	M1
	$u_4 = 2u_3 - 1 = 2(17) - 1 = 33$	Both $u_3 = 17$ and $u_4 = 33$	A1
			[2]
(b)	$\sum_{r=1}^{4} u_r = u_1 + u_2 + u_3 + u_4$		
	$(u_1) = 5$	$(u_1) = 5$	B1
	$\sum_{r=1}^{4} u_r = "5" + 9 + "17" + "33" = 64$	Adds their first four terms obtained legitimately (see notes below)	M1
		64	A1
			[3]
			5 marks
5. (a)	Gradient of l_2 is $\frac{1}{2}$ or 0.5 or $\frac{-1}{-2}$		B1
	Either $y - 6 = "\tfrac{1}{2}"(x - 5)$ or $y = "\tfrac{1}{2}"x + c$ and $6 = "\tfrac{1}{2}"(5) + c \Rightarrow c = ("\tfrac{7}{2}")$.		M1
	$x - 2y + 7 = 0$ or $-x + 2y - 7 = 0$	or $k(x - 2y + 7) = 0$ with ***k* an integer**	A1
			[3]
(b)	Puts $x = 0$, or $y = 0$ in **their** equation and solves to find appropriate co-ordinate		M1
	x-coordinate of A is -7 and y-coordinate of B is $\frac{7}{2}$.		A1 cao
			[2]
(c)	Area $OAB = \frac{1}{2}(7)\left(\frac{7}{2}\right) = \frac{49}{4}$ (units)^2	Applies $\pm\frac{1}{2}$(base)(height)	M1
		$\frac{49}{4}$	A1 cso
			[2]
			7 marks

Question Number	Scheme	Marks
6. (a)		
	$y = \frac{2}{x}$ is translated up or down.	M1
	$y = \frac{2}{x} - 5$ is in the correct position.	A1
	Intersection with x-axis at $\left(\frac{2}{5}, \{0\}\right)$ **only** Independent mark.	B1
	$y = 4x + 2$: attempt at straight line, with positive gradient with positive y intercept.	B1
	Check graph in question for possible answers and space below graph for answers to part (b) Intersection with x-axis at $\left(-\frac{1}{2}, \{0\}\right)$ **and** y-axis at $\left(\{0\}, 2\right)$.	B1 **[5]**
(b)	Asymptotes : $x = 0$ (or y-axis) and $y = -5$. (Lose second B mark for extra asymptotes) An asymptote stated correctly. Independent of (a)	B1
	These two lines only. Not ft their graph.	B1 **[2]**
(c)	Method 1: $\frac{2}{x} - 5 = 4x + 2$ Method 2: $\frac{y-2}{4} = \frac{2}{y+5}$	M1
	$4x^2 + 7x - 2 = 0 \Rightarrow x =$ $y^2 + 3y - 18 = 0 \rightarrow y =$	dM1
	$x = -2, \frac{1}{4}$ $y = -6, 3$	A1
	When $x = -2$, $y = -6$, When $x = \frac{1}{4}$, $y = 3$ When $y = -6$, $x = -2$ When $y = 3$, $x = \frac{1}{4}$.	M1A1 **[5]**
		12 marks
7.	Lewis; arithmetic series, $a = 140$, $d = 20$.	
(a)	$T_{20} = 140 + (20 - 1)(20); = 520$ Or lists 20 terms to get to 520 OR $120 + (20)(20)$	M1; A1 **[2]**
(b)	Method 1 **Either:** Uses $\frac{1}{2}n(2a + (n-1)d)$ Method 2 **Or:** Uses $\frac{1}{2}n(a + l)$	M1
	$\frac{20}{2}\left(2 \times 140 + (20 - 1)(20)\right)$ $\frac{20}{2}\left(140 + "520"\right)$ ft 520	A1
	6600	A1 **[3]**
(c)	Sian; arithmetic series, $a = 300$, $l = 700$, $S_n = 8500$	
	Either: Attempt to use $8500 = \frac{n}{2}(a + l)$ **Or:** May use both $8500 = \frac{1}{2}n(2a + (n-1)d)$ and $l = a + (n-1)d$ **and eliminate** d	M1
	$8500 = \frac{n}{2}(300 + 700)$ $8500 = \frac{n}{2}(600 + 400)$	A1
	$\Rightarrow n = 17$	A1 **[3]**
		8 marks

Question Number	Scheme	Marks
8.	$(\frac{dy}{dx}=)\quad -x^3+"2"x^{-2}-"\left(\frac{5}{2}\right)"x^{-3}$	M1
	$(y=)\quad -\frac{1}{4}x^4+\frac{"2"x^{-1}}{(-1)}-"\left(\frac{5}{2}\right)"\frac{x^{-2}}{(-2)}\ (+c)$ — Raises power correctly on any one term. Any two follow through terms correct.	M1 A1ft
	$(y=)\quad -\frac{1}{4}x^4+\frac{2x^{-1}}{(-1)}-\frac{5}{2}\frac{x^{-2}}{(-2)}\ (+c)$ — This is not follow through – must be correct	A1
	Given that $y=7$, at $x=1$, then $7=-\frac{1}{4}-2+\frac{5}{4}+c \Rightarrow c=$	M1
	So, $(y=)\quad -\frac{1}{4}x^4-2x^{-1}+\frac{5}{4}x^{-2}+c,\ c=8$ **or** $(y=)-\frac{1}{4}x^4-2x^{-1}+\frac{5}{4}x^{-2}+8$	A1
		[6]
		6 marks
9. (a)	**Method 1:** Attempts b^2-4ac for $a=(k+3)$, $b=6$ and their c. $c\neq k$	M1
	$b^2-4ac=6^2-4(k+3)(k-5)$	A1
	$(b^2-4ac=)\quad -4k^2+8k+96$ or $-(b^2-4ac=)\quad 4k^2-8k-96$ (with no prior algebraic errors)	B1
	As $b^2-4ac>0$, then $-4k^2+8k+96>0$ and so, $k^2-2k-24<0$	A1
	Method 2: Considers $b^2>4ac$ for $a=(k+3)$, $b=6$ and their c. $c\neq k$	M1
	$6^2>4(k+3)(k-5)$	A1
	$4k^2-8k-96<0$ or $-4k^2+8k+96>0$ or $9>(k+3)(k-5)$ (with no prior algebraic errors)	B1
	and so, $k^2-2k-24<0$ following correct work	A1
		[4]
(b)	Attempts to solve $k^2-2k-24=0$ to give $k=$ $(\Rightarrow$ Critical values, $k=6,-4.)$	M1
	$k^2-2k-24<0$ gives $-4<k<6$	M1 A1
		[3]
		7 marks

Question Number	Scheme	Marks
10. (a)	This may be done by completion of square or by expansion and comparing coefficients	
	$a = 4$	B1
	$b = 1$	B1
	All three of $a = 4$, $b = 1$ and $c = -1$	B1
		[3]
(b)	 U shaped quadratic graph.	M1
	The curve is correctly positioned with the minimum in the third quadrant. . It crosses x axis twice on negative x axis and y axis once on positive y axis.	A1
	Curve cuts y-axis at $(\{0\}, 3)$.only	B1
	Curve cuts x-axis at $\left(-\frac{3}{2}, \{0\}\right)$ and $\left(-\frac{1}{2}, \{0\}\right)$.	B1
		[4]
		7 marks
11.	$C: y = 2x - 8\sqrt{x} + 5, \quad x \ldots 0$	
(a)	So, $y = 2x - 8x^{\frac{1}{2}} + 5$	
	$\frac{\mathrm{d}y}{\mathrm{d}x} = 2 - 4x^{-\frac{1}{2}} + \{0\} \qquad (x > 0)$	M1 A1 A1
		[3]
(b)	(When $x = \frac{1}{4}$, $y = 2\left(\frac{1}{4}\right) - 8\sqrt{\left(\frac{1}{4}\right)} + 5$ so) $\quad y = \frac{3}{2}$	B1
	(gradient $= \frac{\mathrm{d}y}{\mathrm{d}x} =$) $2 - \frac{4}{\sqrt{\left(\frac{1}{4}\right)}} \quad \{= -6\}$	M1
	Either : $y - "\frac{3}{2}" = "-6"\left(x - \frac{1}{4}\right)$ **or**: $y = "-6"x + c$ and $"\frac{3}{2}" = "-6"\left(\frac{1}{4}\right) + c \Rightarrow c = "3"$	M1
	So $\underline{y = -6x + 3}$	A1
		[4]
(c)	Tangent at Q is parallel to $2x - 3y + 18 = 0$	
	$\left(y = \frac{2}{3}x + 6 \Rightarrow\right)$ Gradient $= \frac{2}{3}$. so tangent gradient is $\frac{2}{3}$	B1
	So, $"2 - \frac{4}{\sqrt{x}}" = "\frac{2}{3}"$ Sets their gradient function = their numerical gradient.	M1
	$\Rightarrow \frac{4}{3} = \frac{4}{\sqrt{x}} \Rightarrow x = 9$ Ignore extra answer $x = -9$	A1
	When $x = 9$, $y = 2(9) - 8\sqrt{9} + 5 = -1$ Substitutes their found x into equation of curve.	M1
	$y = -1$.	A1
		[5]
		12 marks